

The ζ Coordinate Framework: Normalized-to-Analytic Conversion and $\beta - \eta$ Parity Decomposition

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Abstract

We unify the primary ζ coordinate framework with the $\beta - \eta$ parity difference decomposition. Table 1 defines the conversion between normalized integer coordinates and continuous analytic L-function evaluations across the ζ Parent Series, the Even Parity Sector, and the Odd Parity Sector via the scale $4^n/\zeta$, yielding the Coordinate Ratio $K(n)$. Table 2 isolates the parity difference sector to extract the Coordinate Remainder $k(n) = K(n) - 2^n$.

Table 1. The ζ Coordinate Framework: Normalized-to-Analytic Conversion via the Scale $4^n/\zeta$

Normalized Values for $\zeta(4)$ with Associated Formulas

Even Parity ζ		Parent Series ζ		Odd Parity ζ	
8	7	128	112	120	116.9566528
15	1	240	16	236.9566528	3.043347237
16	14	256	224	240	233.9133055
	224		57344		56139.19333
	14.96662955		239.4660728		236.9371084
Coordinate Ratio $K(4) = \frac{233.9133055}{14} = 16.70809325$					
2^{n-1}	$2^{n-1} - 1$	$2^{n-1} \cdot 2^n$	$2^{n-1}(2^n - 2)$	$2^n \cdot 2^{n-1} - 2^{n-1}$	$(2^{n-1} - 1)(2^n + k(n))$
$2^n - 1$	1	$2^n \cdot 2^n - 2^n$	2^n	$2^{n-1}(1 + k(n)(1 - 2^{-(n-1)}))$	$2^{n-1}(1 - k(n)(1 - 2^{-(n-1)}))$
2^n	$2^n - 2$	$2^n \cdot 2^n$	$2^n(2^n - 2)$	$2^n(2^n - 1)$	$(2^n - 2)(2^n + k(n))$
	$2^n(2^n - 2)$		$2^n \cdot 2^n \cdot 2^n(2^n - 2)$		$2^n(2^n - 1)(2^n - 2)(2^n + k(n))$
	$(2^n(2^n - 2))^{1/2}$		$2^n(2^n(2^n - 2))^{1/2}$		$(2^n(2^n - 1)(2^n - 2)(2^n + k(n)))^{1/2}$
Coordinate Ratio $K(n) = \frac{(2^n - 2)(2^n + k(n))}{2^n - 2} = 2^n + k(n)$					

Analytic Values for $\zeta(4)$ with Associated Formulas

Even Parity ζ		Parent Series ζ		Odd Parity ζ	
0.033822601	0.029594776	0.541161617	0.473516415	0.507339016	0.494472276
0.063417377	0.004227825	1.014678032	0.067645202	1.001811292	0.012866740
0.067645202	0.059189552	1.082323234	0.947032829	1.014678032	0.988944552
	0.004003889		1.024995634		1.003460311
	0.063276293		1.012420681		1.001728661
Coordinate Ratio $K(4) = \frac{0.988944552}{0.059189552} = 16.70809325$					
$\frac{1}{2}4^{-n}(\zeta(n, \frac{2}{4}) + \zeta(n, \frac{4}{4}))$	$\frac{1}{2}4^{-n}(\zeta(n, \frac{2}{4}) - \zeta(n, \frac{4}{4}))$	$\frac{1}{2}\zeta$	$\frac{1}{2}\eta$	$\frac{1}{4}(\zeta + \eta)$	$\frac{1}{4}(\beta + \beta)$
$4^{-n}\zeta(n, \frac{2}{4})$	$4^{-n}\zeta(n, \frac{4}{4})$	$\frac{1}{2}(\zeta + \eta)$	$\frac{1}{2}(\zeta - \eta)$	$\frac{1}{4}((\zeta + \eta) + (\beta + \beta))$	$\frac{1}{4}((\zeta + \eta) - (\beta + \beta))$
$4^{-n}(\zeta(n, \frac{2}{4}) + \zeta(n, \frac{4}{4}))$	$4^{-n}(\zeta(n, \frac{2}{4}) - \zeta(n, \frac{4}{4}))$	ζ	η	$\frac{1}{2}(\zeta + \eta)$	$\frac{1}{2}(\beta + \beta)$
	$\eta \cdot 4^{-n}\zeta$		$\zeta \cdot \eta$		$\frac{1}{2}\beta(\zeta + \eta)$
	$(\eta \cdot 4^{-n}\zeta)^{1/2}$		$(\zeta \cdot \eta)^{1/2}$		$(\frac{1}{2}\beta(\zeta + \eta))^{1/2}$
Coordinate Ratio $K(n) = \frac{\frac{1}{2}(\beta(n) + \beta(n))}{4^{-n}(\zeta(n, 2/4) - \zeta(n, 4/4))} = \frac{\beta(n)}{2^{-n}\eta(n)} = 2^n \frac{\beta(n)}{\eta(n)}$					

Table 2. The $k(n)$ Coordinate Remainder: $\beta - \eta$ Parity Difference Decomposition

Normalized Values for $\zeta(4)$ with Associated Formulas

Even Parity ζ		Odd Parity $(\beta - \eta)$	
8	7	8	4.956652763
15	1	12.95665276	3.043347237
16	14	16	9.913305527
	224		158.6128884
	14.96662955		12.59416089
Coordinate Remainder $k(4) = \frac{9.913305527}{14} = 0.70809325$			
2^{n-1}	$2^{n-1} - 1$	2^{n-1}	$(2^{n-1} - 1)k(n)$
$2^n - 1$	1	$2^{n-1}(1 + k(n)) - k(n)$	$2^{n-1}(1 - k(n)) + k(n)$
2^n	$2^n - 2$	2^n	$(2^n - 2)k(n)$
	$2^n(2^n - 2)$		$2^n(2^n - 2)k(n)$
	$(2^n(2^n - 2))^{1/2}$		$(2^n(2^n - 2)k(n))^{1/2}$
Coordinate Remainder $k(n) = \frac{(2^n - 2)k(n)}{2^n - 2} = k(n)$			

Analytic Values for $\zeta(4)$ with Associated Formulas

Even Parity ζ		Odd Parity $(\beta - \eta)$	
0.033822601	0.029594776	0.033822601	0.020955861
0.063417377	0.004227825	0.054778462	0.012866740
0.067645202	0.059189552	0.067645202	0.041911722
	0.004003889		0.002835127
	0.063276293		0.053245910
Coordinate Remainder $k(4) = \frac{0.041911722}{0.059189552} = 0.70809325$			
$\frac{1}{2}4^{-n}(\zeta(n, \frac{2}{4}) + \zeta(n, \frac{4}{4}))$	$\frac{1}{2}4^{-n}(\zeta(n, \frac{2}{4}) - \zeta(n, \frac{4}{4}))$	$\frac{1}{4}(\zeta - \eta)$	$\frac{1}{4} \cdot 2(\beta - \eta)$
$4^{-n}\zeta(n, \frac{2}{4})$	$4^{-n}\zeta(n, \frac{4}{4})$	$\frac{1}{4}((\zeta - \eta) + 2(\beta - \eta))$	$\frac{1}{4}((\zeta - \eta) - 2(\beta - \eta))$
$4^{-n}(\zeta(n, \frac{2}{4}) + \zeta(n, \frac{4}{4}))$	$4^{-n}(\zeta(n, \frac{2}{4}) - \zeta(n, \frac{4}{4}))$	$\frac{1}{2}(\zeta - \eta)$	$\frac{1}{2} \cdot 2(\beta - \eta)$
	$\eta \cdot 4^{-n}\zeta$		$\frac{1}{2}(\zeta - \eta)(\beta - \eta)$
	$(\eta \cdot 4^{-n}\zeta)^{1/2}$		$(\frac{1}{2}(\zeta - \eta)(\beta - \eta))^{1/2}$
Coordinate Remainder $k(n) = \frac{\beta(n) - \eta(n)}{4^{-n}(\zeta(n, 2/4) - \zeta(n, 4/4))} = \frac{\beta(n) - \eta(n)}{2^{-n}\eta(n)} = 2^n \left(\frac{\beta(n)}{\eta(n)} - 1 \right)$			